

EW\_IDO\_D06\_PTO Preliminary design report\_v0

Table of Contents

Complement with list of figures and/or list of tables where appropriate

[Table of Contents 1](#_Toc109286362)

[3. Improved Power Take Off design 2](#_Toc109286363)

[3.1. Preliminary design tool 2](#_Toc109286364)

[3.1.1. Input parameters 2](#_Toc109286365)

[3.1.2. BEM model 4](#_Toc109286366)

[3.1.3. Actuator-disk model 12](#_Toc109286367)

[3.1.4. Stochastic analysis 14](#_Toc109286368)

[3.2. Aerodynamic optimization procedure 16](#_Toc109286369)

[3.3. Preliminary design results 2](#_Toc109286370)

[Annex 2- Genetic algorithm optimization tool description 7](#_Toc109286371)

1. Improved Power Take Off design

The power take off design described in (ref), although valid for performing calculations on the global behaviour of the turbine, leaves room for further improvements. In particular, such a model does not consider the geometrical specificities of the turbine blades or the impact of their aerohydrodynamic behaviour on the energetic outcome of the system. Developing a design tool that accounts for such geometrical and aerohydrodynamic considerations constitutes a relevant step forward. Subsection 3.1 deals with the fundamentals of such a design tool. Having implemented the design algorithm, Subsection 3.2 introduces the optimization procedure employed for leveraging among the set of potential designs, describing the cost or fitness functions that are used for scoring the different turbine configurations. Finally, Subsection 3.3 presents a number of preliminary results stemming from the application of the procedures explained in Subsections 3.1 and 3.2 subjected to a set of design constraints.

* 1. Preliminary design tool

The main purpose of improving the power take off design is to own an analytical tool capable of providing the energetic outcome of a generic Wells turbine parting from a minimal set of design parameters that determine such a turbine’s aerohydrodynamic behaviour.

On this respect, developing an appropriate aerohydrodynamic model based on the working principles of a Wells turbine comprises the main task of this subsection. The simplest analytical model that accounts for the aerohydrodynamic behaviour of a Wells turbine is based on the two-dimensional cascade theory, or the blade-element momentum (BEM) approach (cite). Starting with such a model, which is briefly described in what comes, the preliminary design tool is developed by complementing the two-dimensional approach with the actuator-disk theory (cite). The following lines provide a short introduction into the fundamentals of both approaches, which nonetheless suffice for grasping the idea underlying the design tool.

* + 1. Input parameters

However, before proceeding with the models themselves, it is deemed necessary to introduce the basic nomenclature that is employed throughout the text for addressing both the physical and geometrical parameters that determine the aerohydrodynamic behaviour of a Wells turbine. For the sake of conciseness, the analytical development is constrained to a monoplane Wells turbine, meaning that the considered system owns a single blade-cascade or stage. Nevertheless, the development is extendible to an arbitrary number of stages. With all, the main geometrical parameters of a Wells turbine are depicted on Figure 1(a) and (b).

Figure 1(a) shows the isometric view of the Wells turbine, focusing on the section of the system where the monoplane stage is located. Given the shape of the device, it is practical to adopt a cylindrical coordinate system. As shown in the figure:

* The axis runs along the axis of the turbine, and is named the axial direction.
* The axis travels along the radius of the turbine and the blades, and is named the radial direction.
* The axis is circumferential, and is termed the tangential direction.

Besides the coordinate system, Figure 1(b) shows the front view of the turbine, which serves the purpose of defining the geometrical parameters that determine the system. The relevant variables are:

* : the radii of the turbine’s hub, tip and casing, respectively.
  + The ratio between the hub and tip radii is termed the hub-to-tip-ratio, namely .
  + The reason for defining both the tip and casing radii stems from the gap that exists between them, which is known as the tip clearance, .
* Solidity : it is a geometrical parameter that depends on two other geometrical variables itself, i.e. , where:
  + : addresses the chord of the blades, or the leading-to-trailing-edge distance, and is a function of the radial parameter, i.e. .
  + : refers to the distance between blades, namely the one between adjacent leading- or trailing-edges, and also depends on the radius, i.e. .

from which it is deduced that the solidity also varies radially, .

Figure 1: Schematic of a Wells turbine's configuration; (a) isometric view; (b) front view; (c) two-dimensional cascade for the BEM approach.

* : the number of turbine blades.
* : the angular velocity of the turbine, in rad/sec. When it is given in rpm units, the employed symbol is .

The set of parameters constitutes a minimal collection of variables required for computing the aerodynamic behaviour of a generic Wells turbine. In fact, it is necessary to include an additional dependency, which can be observed in Figure 1(c); indeed, such a figure shows the monoplane stage represented in a linear layout, which is the approach adopted by the BEM model. When depicted thusly, the blades are viewed in their cross-sectional plane, showing the particular geometry they own. Such a cross-sectional shape is termed the airfoil geometry of the blades.

Due to the self-rectifying nature of the Wells turbine, it is usual to employ symmetric airfoils for achieving similar behaviours regardless of the airflow direction. Among the set of possibilities for the symmetric airfoils, the denomination addresses a family of morphologies that are of standard use within the turbomachinery industry, mainly due to their normalized features and well-known aerodynamic properties (cite). Such a denomination comprises the characters followed by a set of four digits, each of which determines a specific geometrical property of the airfoil. Wells turbine configurations employing the , or airfoils are typical (cite), as well as designs with blades owning a radially varying profile, i.e. .

Considering that the airflow travels around the blades, passing across the profiles from their leading- to trailing-edges, it is deduced that the specific shape of the airfoils is aerodynamically relevant for determining the energetic outcome of a turbine design. Indeed, the loads that are exerted upon the airfoil, which ultimately cause the torque that sets the turbine into motion and produce energy, depend on the cross-sectional shape of the blades. Thus, such a shape sums up to the additional geometrical parameter that is required as an input. Without loss of generality, and saving the generic nature of the description made so far, the airfoil shapes of the blades are represented by the expression , meaning that they constitute profiles liable to vary in the radial direction. Hence, the set of parameters that serve the purpose of geometrical input is:

|  |  |  |
| --- | --- | --- |
|  | , | Eq. 1 |

It is to notice that such a collection is not immutable, but that it can be replaced by an alternative set of parameters insofar it provides the same information content, such as discarding and and providing and instead, for instance.

Apart from the geometrical parameters, it is required to establish the flow conditions at the entrance of the system. This is achieved by providing two intensive variables for the incoming air, i.e. the couple representing the ambient pressure and temperature respectively, which fix the thermodynamic state of the fluid. Besides, it is mandatory to provide a surrogate for the kinematic state of the flow, which is usually determined by setting the absolute velocity vector at the entrance, namely . Such a notation means that, in a generic case, both its magnitude and direction are dependent on the radial parameter. With all, the flow-related input to the system comprises:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 2 |

Thus, the parametrical input to the system can be summarized as:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 3 |

* + 1. BEM model

The cross-sectional monoplane cascade depicted in Figure 1(c), apart from providing insights into the geometry- and flow-related input parameters, is the baseline for introducing the BEM approach. The main idea underlying the BEM procedure is that it is possible to divide a Wells turbine’s blade into a portion of infinitesimal elements, as represented by the bottom-left blades in Figure 1(b). Those elements cover an infinitesimal radial dimension of , being the number of elements set in advance by the user. The larger its value, the smaller the elements become, and the finer the analysis turns, although its computational cost rises accordingly. Finding a trade-off between the accuracy of the results and the computational cost is mandatory for establishing a well-suited value for .

**Model hypotheses:**

The splitting of the blade on a set of infinitesimal blade-elements is doable under a number of different assumptions as for the dynamics of the flow. In particular, the BEM approach assumes that the airflow passing through the cascade is:

* Steady: i.e. the temporal variations within the flowfield are negligible.
* Incompressible: which means that the Mach numbers are small enough (the magnitude of the velocity is small when compared to the speed of sound), resulting in negligible compressibility effects and a constant density of the fluid.
* Irrotational: which states that vorticity variations are negligible.
* Axisymmetric: allowing to reduce the analysis to the tandem-like arrangement depicted on Figure 1(c).
* Radially balanced, or in radial equilibrium: meaning that the momentum balance in the radial direction expresses that the pressure force equals the centripetal acceleration.

Those assumptions, although result in an overly simplistic picture of the flowfield, allow performing the aerohydrodynamic analysis in an affordable manner, as it turns possible to reduce the set of equations required for computing the energetic outcome of the turbine to an algebraic system. Besides, the BEM approach constitutes the baseline for developing the actuator-disk theory, which discards the radial equilibrium condition and leads towards a qualitatively closer model to real conditions. In a nutshell, a steady, incompressible, irrotational, axisymmetric and radially-balanced flow can be represented as in Figure 2: radial velocities are allowed just in the monoplane cascade, with the streamlines comprising concentric cylinders both upstream and downstream the blades.



Figure 2: Schematic of the radial equilibrium notion; no radial velocity components are allowed outside the monoplane.

**Velocity triangles and aerodynamic loads:**

The determination of the energetic outcome of a generic Wells turbine on the basis of the BEM approach begins from analysing the near-flowfield of an isolated blade. Parting from the sketch in Figure 1(c), the analysis of a single blade-element leads to the configuration shown in Figure 3.



Figure 3: BEM approach when applied to a single blade

There are three relevant sets of variables on the figure, which can be summarized as the upstream kinematic set, the dynamic set, and the downstream kinematic set. The aim of the BEM approach is to provide the three sets in terms of the input variables:

* Upstream kinematic set:

It addresses the kinematic states of the fluid both upstream and downstream the considered blade element. Such kinematic states are represented by three velocity components, which constitute the so-called velocity triangles. The upstream and downstream stages are to be understood as located at an infinitesimal distance away from the airfoil, with such a distance being augmented on the figure for illustrative purposes.

The velocity components of the upstream stage are particularized by a (1) subscript, indicating that they correspond to the flow before passing through the monoplane cascade. The downstream ones are specified by a (2) subscript. The (i) subscript present in the overall set of velocity components stands for naming the generic blade element considered in the analysis, and is a dummy index that runs over .

The or subscripts in some of the velocity components represent projections of the corresponding velocity vectors upon the axial or tangential axes, respectively.

The components of the velocity triangles are understood, hence, as:

* + - : represent the absolute velocities of the fluid upstream and downstream the blade element. Such velocities are the ones observable from an inertial reference frame within which the monoplane cascade would own a rotary motion. The components and are the projections upon the axial and tangential axes of the absolute velocities, respectively. The angles between the directions of and the tangential axes are represented by , and are known as the absolute angles of the configuration.
    - represents the tangential velocity at the specific blade-element considered, and does not vary when passing from the upstream to the downstream velocity triangle.
    - : stand for the relative velocity components, which are the ones that would be observed in a non-inertial reference frame owning the same rotational speed as the monoplane cascade. The axial components  coincide with , given that runs along the axis. The tangential components show the values required for closing the velocity triangles. The angles between the directions of and the tangential axes are represented by , and are known as the relative angles of the configuration.

On practical grounds, the change in relative velocity when passing from the upstream to the downstream stage, i.e. , is the working principle by which the Wells turbine produces a net energetic outcome. Thus, determining the velocity triangles is an essential task of the BEM approach. Functionally speaking, the unknown velocity components of the upstream velocity triangle present in Figure 3 can be written in terms of the input variables via geometrical identities and relationships. Given that are specified for the considered blade element, and that is provided as an input variable, thus determining the tangential velocity , naming to the set :

|  |  |  |
| --- | --- | --- |
|  | , | Eq. 4 |

where the different equalities in the equation are equivalent in terms of functional dependencies, considering that the input variables other than do not alter such functional relationships and that, accordingly, the triad or the duo can be generalized to .

The determination of the downstream velocity triangle requires introducing the second relevant aspect present in Figure 3.

* Dynamic set:

It refers to the loads that the flow exerts upon the airfoil. On a non-inertial reference frame rotating with the same angular velocity as the monoplane cascade, the airfoil gets subjected to the upstream relative velocity , which impinges the blade-element with an incidence angle of , turning such an angle to the angle-of-attack of the airfoil. Within the paradigm of the non-inertial reference frame, the loads upon the airfoil get decomposed into:

* + - : with the symbol representing the differential character of the load, and the subscript standing for ‘lift’, which acts perpendicular to the direction of the relative flow. The superscript refers to the fact that those loads are obtained for a single-blade configuration.
    - : with the subscript standing for ‘drag’, which represents the aerodynamic drag that the airfoil experiences along the same direction of the relative flow.

When considering an inertial frame that does not rotate with the monoplane cascade, the lift and drag loads can be projected upon the axial and tangential axes, yielding:

* + - : represents the axial force component, and is related to the pressure loss that ensues across the monoplane cascade.
    - : stands for the tangential force component, and is the driving mechanism of the Wells turbine, causing the momentum or torque that makes the monoplane cascade rotate, thus producing a net power output.

The BEM approach allows relating the lift and drag loads to the input variables through the relative angle and the velocity computed through Eq. 4 (cite). In fact, the determination of relies on:

* + - The Reynolds number of the flow, which is a dimensionless number defined as , where stand for the air’s density and viscosity, respectively, and can be computed through the input thermodynamic state fixed by .
    - The relative angle-of-attack, .
    - The geometry of the airfoil, . For a given geometry, the so-called characteristic curves of an airfoil are the -based family-curves that represent the lift and drag loads in terms of the angle-of-attack, namely:

|  |  |  |
| --- | --- | --- |
|  | , | Eq. 5 |
|  |  | Eq. 6 |

Usually, the functional relations in Eq. 5 and Eq. 6 are synthesized in what is known as the polar of the airfoil, or its relationship. Naming it as , and subsuming the dependency within the set of variables:

|  |  |  |
| --- | --- | --- |
|  | , | Eq. 7 |

which shows that the determination of the loads derived straightforwardly from the input parameters. The same can be said of , as their derivations from the variables only require a projection based on the angle-of-attack:

|  |  |  |
| --- | --- | --- |
|  | , | Eq. 8 |
|  |  | Eq. 9 |

The bottomline is that the computation of the axial and tangential loads depends exclusively on the set of input parameters defined in Eq. 3. Synthesizing the dynamic set by naming the of loads as :

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 10 |

* Downstream kinematic set:
  + Once the determination of the aerodynamic loads is performed, the BEM approach allows computing the downstream velocity triangle, which completes the monoplane cascade calculations upon a single blade. Naming to the set , the generic functional relation reads:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 11 |

which, as mentioned, closes the task of formulating the calculations of the upstream velocity triangle and the aerodynamic loads in terms of the input set of variables, whereas the downstream triangle depends, additionally, on the imposition of the radial equilibrium condition, generically represented by .

**Interference factor:**

The expressions provided in Eq. 4, Eq. 10 and Eq. 11 are deduced from the aerodynamic analysis of a single blade, as shown in Figure 3. However, the sketch in Figure 1(c) represents a tandem-like arrangement, which corresponds to the linear monoplane-cascade of a Wells turbine. The shift from a single-blade analysis to the multiple-blade monoplane-cascade requires introducing an interference factor on the BEM approach developed so far. The purpose of such a factor is to preserve the functional relations deduced in the single-blade analysis, so that performing an additional BEM approach upon a multiple-blade configuration is avoided. The interference factor is a corrective parameter, which gets applied upon those variables of the BEM formulation that are affected by the inclusion of the remaining blades.

There are several approaches for modelling the interference factor. Two common ways are to model it analytically, by relating it to the geometry- and flow-related parameters that a two-dimensional BEM theory predicts the interference factor depends on (cite), or to model it empirically via a functional relation obtained from experimental tests (cite) or CFD simulations. In either case, the factor is known to depend on the geometrical aspects of the Wells design, which are considered by the input set of parameters. The approach adopted herein is the empirical one, which relates the interference factor, named , to the solidity of the blade-element at each radial stage (cite):

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 12 |

The interference factor affects the lift and drag loads and, consequently, their axial and tangential projections. The corrected differential forces, namely the duo (notice the absence of the superscript), become:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 13 |
|  |  | Eq. 14 |

which remain as exclusive functions of the input parameters. The loads and downstream velocity triangles computed for a multiple-blade monoplane-cascade turn, thus:

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | Eq. 15 |
|  |  | | Eq. 16 |

which completes the multiple-blade monoplane-cascade calculation task, as the overall set of parameters is referenced to the input set of variables.

**Energetic outcome:**

The determination of the three sets of parameters in terms of the input variables, namely the upstream and downstream velocity triangles and the aerodynamic loads, sets forth the way towards the computation of the energetic outcome of the turbine based on the same input parameters. Indeed, the angular-momentum-conservation equation provides the blade-element-related differential torque and power (cite):

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 17 |
|  |  | Eq. 18 |

where the number of blades, namely , appears in Eq. 17 as the torques provided by each single blade are summed up, and the differential power is obtained by simply multiplying such a torque by the rotational speed of the system.

Besides, the momentum-conservation equation in the axial direction, applied within the upstream and downstream stages, provides the static-to-static pressure-loss produced by the passing of the flow across the monoplane-cascade:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 19 |

This last expression can be turned into a total-to-total pressure-loss , by considering the kinetic energy balance between the upstream and downstream stages:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 20 |

This total-to-total pressure-loss can be understood as a surrogate for the input or available power of the system , namely:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 21 |

with being the differential flow-rate at the considered blade-element. Considering that the power produced by the turbine is the output power, i.e. , the differential efficiency can be defined as the ratio between the output and input powers:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 22 |

which closes the analytical derivation provided by the BEM approach. The overall torques, powers, pressure-losses and efficiencies can be obtained by integrating Eq. 17, Eq. 18, Eq. 20 and Eq. 22 along the radial direction, respectively.

**Radial equilibrium condition:**

As they stand, the equations Eq. 20 and Eq. 22 depend on the imposition of the radial equilibrium condition in addition to the set of input variables. Closing the system of equations requires formulating such a condition in terms of the input variables. Imposing the radial equilibrium provides a differential equation that relates the absolute downstream axial velocity with its tangential counterpart, i.e. (cite):

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 23 |

Eq. 23 is not directly computable. Instead, it requires introducing an iterative loop within the otherwise algebraic system of equations. Such an iterative procedure asks for a convergence criterion, which is set to obtaining the equality between the computed values of the upstream and downstream flow-rates, thus ensuring the continuity equation. It is to notice that, insofar Eq. 23 expresses a differential relationship between absolute velocity components along the radial direction, such components (and, hence, the absolute velocity itself) will depend on the radial parameter, i.e. . Enforcing the continuity equation results in the expression:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 24 |

On practical grounds, the convergence criteria is set to lie below a predefined threshold, named , with the condition being expressed as:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 25 |

As such, the iterative procedure is performed as follows:

* The downstream triangle is solved with .
* is obtained.
* The radial equilibrium condition (Eq. 24) is enforced: is obtained.
* A continuity checking (Eq. 25) is performed. In case Eq. 25 is not fulfilled, the downstream triangle is re-computed, which closes the cycle.

After achieving continuity convergence, the velocity triangles that correspond to the blade elements of the considered turbine stage have been resolved under the radial equilibrium condition. The continuity-enforcing-loop allows imposing the radial equilibrium condition and discarding its explicit dependency on the downstream velocity components and energetic outcome variables, making the system of equations exclusively dependent on the input set of variables.

**Dimensionless formulation:**

The formulation described so far is expressed in terms of dimensional variables, as is readily observed when analysing Eq. 1 to Eq. 25. A common practice in aerohydrodynamic models and analyses is to recast those dimensional variables into dimensionless magnitudes. Making the formulation dimensionless allows discarding magnitude- and scale-related dependencies, generalizing the model and turning it applicable to generic systems, which can be later compared on the basis of those dimensionless magnitudes (cite).

Such dimensionless magnitudes have already shown up during the description of the model. The Reynolds number, introduced for showing the functional dependencies of the differential loads (Eq. 5 and Eq. 6), is one of them. The efficiency of the system, expressed in Eq. 22, is another one. However, the inclusion of those dimensionless magnitudes does not result in an overall dimensionless formulation of the model. For such a purpose, and according to Buckingham’s theory of dimensional analysis (cite), it is necessary to turn dimensionless three input magnitudes. The commonest practice in Wells-related turbomachinery applications is to employ the so-called flow-, pressure- and power-coefficients as the main dimensionless variables, which are expressed, respectively, as:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 26 |
|  |  |  |
|  |  | Eq. 27 |

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 28 |

where is the tip diameter of the turbine, and and refer to the radially-integrated values of the total-to-total pressure-loss and power output, respectively. The aerodynamic loads are turned into the so-called load coefficients by making them dimensionless thusly:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 29 |

where the subscript being a dummy index that can adopt any of the values, with the corresponding dimensionless magnitude being the lift, drag, axial or tangential coefficient, respectively. With such a formulation, the upstream and downstream velocity triangles can be written in terms of the flow-coefficient, and the loads and energetic outcomes get expressed as functions of . Thus, the outcome of the BEM approach are a set of characteristic curves, which are usually provided in terms of the dimensionless pressure coefficient, i.e. , and . A summary flowchart of the BEM algorithm described so far is shown in Figure 4, where the primed variables (such as ) indicate that they have been made dimensionless.

Figure 4: Flowchart showing the BEM loop

Load calculation

input

No

Yes

Convergency

check

, ,

Output characteristic dimensionless curves

**BEM loop**

Turn dimensionless

Upstream triangle calculation

Downstream triangle calculation

* + 1. Actuator-disk model

The two-dimensional BEM model described in 3.1.2 has the shortcoming of providing an overly simplistic view of the flowfield developed along a Wells turbine. Such a shortcoming stems from the restrictive hypotheses that the model relies on, especially on the radial equilibrium condition that enforces the streamlines to travel in the form of concentric cylinders outside the monoplane-cascade region, as shown in Figure 2.

Empirical results show that the departure from the radial equilibrium condition is not negligible, requiring to introduce such an effect somehow (cite). On this respect, the actuator-disk formulation is the simplest way of modelling the radial variation of the flow both upstream and downstream the monoplane region.

As opposed to the two-dimensional BEM theory, the axial component of the velocity is not necessarily constant in an actuator-disk formulation, neither in the axial nor in the radial directions upstream and downstream the disk. In fact, the actuator-disk is modelled as an infinitesimally thin membrane located at the mid-chord stage of the monoplane-cascade. The disk does not perturb the mass-flow rate across it, which consequently is let to vary throughout the entire flowfield. However, it does introduce a discontinuity on the energy-related variables of the configuration, such as the enthalpy , when the fluid passes through the disk. Figure 5 sketches the notion of the actuator disk, which discards the blade in favour of an infinitesimally thin membrane and allows for the radial variation of the mass flow-rate in opposition to the schematic shown in Figure 2, modelling the change in enthalpy through the disk via a step-like jump.



Figure 5: Schematic of the actuator-disk notion; radially-varying mass flow-rates are allowed throughout the flowfield, with energetic parameters being subjected to an abrupt jump across the disk.

The straightforward manner to consider the redistribution of mass-flow is to apply the conservation of the axial momentum and equate it to the force exerted by the flow on the actuator-disk:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 30 |

where addresses the axial force, is the mass flow-rate, stands for the velocity at the disk that corresponds to the blade-element, and refers to the differential area filled by such an element.

Computing the velocity merely requires applying the conservation of energy, equating the axial power acting on the disk to that lost by the flow:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 31 |

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 32 |

Matching Eq. 31 and Eq. 32 means that the kinetic energy extracted from the fluid corresponds to the energy absorbed by the disk. It follows that the velocity at the disk can be expressed as:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 33 |

The implementation of the actuator-disk formulation introduces yet another iterative procedure in the model, which encompasses the two-dimensional BEM theory within. The convergence criterion in the case of the improved model is performed in terms of conservation of energy. The energy outcome from a given computation is compared to the value that the model yielded in the previous iteration , and the iterative loop continues until such a difference falls below a predefined threshold, namely :

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 34 |

The workflow of the iterative procedure goes as follows:

* The two-dimensional BEM theory is applied for resolving the velocity triangles and energetic outcomes of a turbine system.
* The velocity at the disk is computed.
* The Reynolds numbers and load coefficients are recomputed based on the velocity at the disk.
* The two-dimensional BEM theory is reapplied for resolving the velocity triangles and energetic outcomes.
* An energy checking is performed on the outcome power. In case it is not fulfilled, the disk velocity is recomputed, which closes the cycle.

After achieving energy convergence, the velocity triangles that correspond to the blade elements of the considered turbine have been resolved under both the radial equilibrium condition and the actuator-disk condition. The corresponding flowchart is shown in Figure 6.

Figure 6: Flowchart showing the actuator-disk loop

BEM

input

No

Yes

Convergency

check

, ,

Output characteristic dimensionless curves

**BEM\_AD loop**

BEM

Disk velocity calculation

Outcome energy,

Outcome energy,

* + 1. Stochastic analysis

The models described in 3.1.2 and 3.1.3 are formulated for outputting the dimensionless characteristic curves of a given turbine configuration when it is subjected to a collection of input conditions, i.e. a range of incoming flow velocities . When performing such an analysis, the actuator-disk model is applied for each of the input parameters, and the final output consists of the dimensionless characteristic curves constituted by the data-points provided by the input parameters.

However, the caveat of such an approach is that it considers the input parameters as fixed values, when it is known that in actual processes the inputs provided by a generic sea-state are better modelled by an stochastic approach. In fact, it is commonplace to regard the sea-surface motion as a Gaussian process, specifically the pressure oscillation that develops along the turbine’s plenum or air chamber (cite). Naming such a pressure and its standard deviation , the probability density function () of it reads:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 35 |

Having defined the variability of the incoming pressure variable, and assuming a constant angular speed , it is possible to compute the average values of the output variables, namely , and , with being the average of the available power . Those averages read:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 36 |

where, on practical grounds, the integral extends between pressure oscillation values for which their occurrence probability is negligible. Turning the formulation dimensionless, Eq. 36 can be written as:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 37 |

where is the dimensionless deviation of the pressure oscillation, and are defined as:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 38 |

Based on the definitions provided in Eq. 37, the stochastic efficiency () is expressed thusly:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 39 |

defining the ratio between the obtained stochastic power and the available one.

The expressions in Eq. 38 and Eq. 39 are the ones employed for comparing the operational behaviour of different Wells turbine configurations in what follows. The stochastic routine is shown in Figure 7.

input

*, ,*

**Stochastic routine**

BEM\_AD

Stochastic calculations

, ,

Stochastic output

Figure 7: Workflow of the stochastic routine

* 1. Aerodynamic optimization procedure

The design tool described in Section 3.1 enables the characterisation of any turbine configuration and geometrical design as a function of pre-determined flow conditions. That way, the design tool can provide with the most relevant performance metrics that can be used in the design of air-turbines. However, the optimisation of the air-turbine involves, on the one hand, the selection of the configuration. In addition to the traditional monoplane design, other configurations such as the biplane, counter-rotating biplane and variable pitch monoplane have been studied (Figures 1-4 illustrates these different configurations).

|  |  |
| --- | --- |
| Diagram  Description automatically generated  Figure 8: Variable-chord monoplane Wells turbine. | Diagram  Description automatically generated  Figure 9: Biplane Wells Turbine. |
| Diagram  Description automatically generated  Figure 10: Counter-rotating biplane Wells turbine. | Diagram, engineering drawing  Description automatically generated  Figure 11: Variable-pitch monoplane Wells turbine |

|  |
| --- |
| Diagram  Description automatically generated  Figure 12: The most relevant design parameters that affect the performance of Wells turbines [1]. |

On the other hand, different geometrical parameters of the turbine, e.g. the number of blades and their airfoil design and chord length, turbine diameter and hub-to-tip ratio, can significantly affect the performance of the Wells turbine in terms of, both its self-starting and power production characteristics. Figure 5 illustrates the main design parameters that should be considered when optimising a Wells turbine.

Finally, since the air-turbine is only an element integrated in the WEC converter, other operational aspects, such as the required damping in the WEC system, the pressure flow that will drive the air-turbine, or its rotational speed can also have a great impact on the performance and, thus, the final design of the air-turbine.

However, considering all these aspects at the same time within the optimisation procedure makes the optimisation process extremely complex. Therefore, the optimisation process followed in this study is divided into two parts. The first part considers the air-turbine isolated, comparing the different possible configurations of Wells turbines and comparing them in terms of their performance based on the stochastic approach. The results of this first stage in the design process provide a preliminary interpretation of the potential of each configuration, which enables the definition of a select group of limited configurations to be further studied. Figure 6 illustrates the response of the different configurations suggested in Figures 1-4, including monoplane Wells turbines of different chord size. The results are provided based on the stochastic approach that provides a deeper insight into the performance of each configuration in a realistic context with highly variable airflows. Results show that the variable-pitch monoplane (red solid line with star markers) provides the highest efficiency peak followed by the varying-chord monoplane (dark blue with square markers) Wells turbine configuration and the biplane counter-rotating turbine (dark green with triangle markers). However, the variable-pitch and counter-rotating turbines show the capacity to maintain a reasonable performance over a broader range of inputs characteristics, which is crucial for OWC devices. In contrast, the varying-chord and variable-pitch configurations show better self-starting characteristics.

|  |
| --- |
| Chart, line chart  Description automatically generated  Figure 13: Performance of different Wells turbine configurations based on the stochastic approach and isolated turbine behaviour. |

Hence, the select group of Wells turbine configurations includes the varying-chord monoplane, the biplane counter-rotating and variable-pitch monoplane Wells turbines. In the second stage of the design process, an advance optimisation approach has been proposed. This approach enables the optimisation of the air turbine, including the configuration and the main geometrical parameters, considering the holistic performance of the WEC. To that end, and due to the large amount of parameters to be considered, the suggested optimisation approach consists on an iterative method based on Genetic Algorithm (GA) techniques. The main design parameters listed in Figure 5 comprise the first input space. Each combination of these parameters stablishes an individual and the different individuals constitute the search space or population. More specifically, the geometrical parameters considered in this first input space are the following:

1. Number of blades ,
2. Solidity at the hub ,
3. Solidity at the tip ,
4. Hub-to-tip ratio ,
5. The aerodynamic profile at different positions of the blade (NACA profile number),
6. Tip clearance ratio ,

In addition, a diameter and a rotational speed are also defined in this first input space, although these parameters are kept fixed throughout the optimisation approach. Because the optimisation is carried out using dimensionless parameters, variations of the rotational speed can be studied later when the performance of the turbines is analysed in each sea-state.

Hence, the GA is run for several generations, each of which updating the previous population, so that evolves towards the optimal individual. That way, through the analysis of the different populations, each individual gets a fitness score () that is considered as the performance metric of the turbine design represented by that individual, and the GA finds the path towards the optimal or near-optimal combination. For a further description of the GA, refer to Annex 2.

One of the key aspects of this optimisation algorithm is the definition of the cost function or fitness score, since the GA filters the different designs based on this score. In fact, it is in the definition of the fitness score where the elements related to the holistic performance of the WEC enter into play. The characteristics of the WEC performance, such as the extra damping inserted by the PTO system (), the admissible range of rotational speed () and the variable pressure flow available in the OWC chamber represented by the standard deviation of the pressure flow (), form the second input space. Note that this second input space provides the mentioned parameters for each relevant sea-state, including the normalised probability of each sea state (). That way, the performance of each turbine arrangement (individual) is evaluated across the whole operational space of the WEC.

The optimisation approach is illustrated in Figure 7 (a), with a variation of the approach for the variable-pitch configuration presented in Figure 7 (b). The two input spaces are shown at the initial stages of the two approaches, where the first input space is shown to feed the BEM code, while the second input space provides the conditions imposed by the system in order to ensure an adequate holistic performance of the WEC.

|  |  |
| --- | --- |
| Diagram  Description automatically generated   1. *Varying-chord monoplane and the counter-rotating biplane Wells turbines* | Diagram  Description automatically generated   1. *Variable-pitch monoplane Wells turbine* |
| Figure 14: Flowchart of the advance optimisation approaches. | |

Hence, the optimisation process of the varying-chord monoplane and counter-rotating biplane turbines is as follows,

1. The GA selects a combination of the different design parameters included in the first input space based on its experience acquired during the evolution of the algorithm,
2. That selected arrangement is evaluated via the BEM tool providing the main non-stochastic dimensionless variables (),
3. The parameter that represents the linear relationship between the dimensionless flow and pressure coefficients is identified, which also holds the linear relationship between and the rotational speed () of the turbine,
4. The required rotational speed () is calculated for a value of imposed by the system to ensure the optimal holistic performance,
5. The validity of is evaluated by corroborating whether it falls between the admissible range of rotational speed
   1. If falls out of , then the efficiency of the turbine corresponding to that sea-state is defined as
6. If falls within the range, the stochastic curves are computed
7. The efficiency of the turbine is defined as the stochastic efficiency for the value of corresponding to the analysed sea-state
8. The final fitness score is given as the combination of the efficiencies of that specific turbine arrangement weighted with respect to the probability of each sea-state.

Once the GA has analysed enough generations to reach a convergence that ensures that the algorithm has found a global maxima, all the different arrangements analysed in the process are ranked in order to select those that provide the highest fitness score.

In the case of the variable-pitch Wells turbine arrangement, the optimisation approach is identical until the fifth step, where the required rotational speed is determined. However, due to the significant impact of the specific rotational speed on the optimisation of the pitch angles and its relevance on the final turbine performance, the process is modified as follows (see Figure 7 (b)):

1. If falls out of , then the efficiency of the turbine corresponding to that sea-state is defined as (Same as in the previous algorithm)
2. If falls within the range, the optimal pitch angles are identified for each flow condition
3. The stochastic curves are computed, including the optimised pitch angles
4. The efficiency of the turbine is defined as the stochastic efficiency for the value of corresponding to the analysed sea-state
5. The final fitness score is given as the combination of the efficiencies of that specific turbine arrangement weighted with respect to the probability of each sea-state.

That way, both optimisation approaches ensure that the turbine arrangement that optimises the holistic performance of the WEC will be found for any turbine configuration, providing the turbine arrangement that maximises the power extraction across the whole operational space and ensures the desired behaviour of the WEC.

* 1. Preliminary design results

The optimisation approach defined in Section 3.2 allows for the coupled WEC-turbine optimisation in the sense that the control force, number of turbines, and the turbine configuration, size and arrangement are optimised simultaneously. Hence, combining different number of turbines integrated in the WEC and outer diameters of these turbines, four different WEC architectures have been analysed, as presented in Table 1. The turbine design is optimised for each of these WEC architectures and turbine configurations.

|  |  |  |
| --- | --- | --- |
|  | 2 turbines | 3 turbines |
|  | 🗸 | 🗸 |
|  | 🗸 | 🗸 |

Table 1: The different WEC architectures studied via the GA-based optimisation approach.

The details about each of these WEC architectures are defined in the second input space of the GA-based optimisation approach, for which the key parameters control damping and the input pressure vary significantly, leading to significantly different turbine designs. Hence, for each WEC architectures and turbine configuration, the GA-based optimisation approach provides the stochastic curves of the optimal turbine arrangement, so that the selection of the final design is carried out in the post-processing by comparing the fitness scores of the different configurations for each WEC architecture.

Figure 8 illustrates the output of the GA-based approach for the three different turbine configurations and the WEC architecture composed by two 1-meter-diameter turbines. The figure illustrates the stochastic efficiency curves of optimal varying-chord monoplane (red dashed line), counter-rotating biplane (black dash-dotted line) and variable-pitch monoplane turbines (blue solid lines) for that specific WEC architecture.

|  |
| --- |
| Figure 15: Stochastic efficiency curves for the three turbine configurations and the WEC architecture composed by two 1-meter-diameter turbines. |

In the case of the varying-chord monoplane and counter-rotating biplane turbines, a single stochastic curve is shown, since the different rotational speed values do not alter the dimensionless curves. In contrast, due to the impact of the rotational speed on the determination of the optimal pitch angles, different efficiency curves are shown for different sea-states. In this case, the efficiency values for each sea-state are obtained from the efficiency curve corresponding to each specific sea-state.

Each of these curves corresponds to the optimal turbine arrangement, which provides the optimal values of the parameters defined in the first input space of the GA-based optimisation approach. Hence, the optimal geometrical parameters of the turbine corresponding to the WEC architecture with two 1-meter-diamter turbines are shown in Table 2. For the sake of clarity, only the main geometrical parameters are provided, which are identified to be the number of blades (), the solidities () and the hub-to-tip ratio ():

Table 2: Optimal geometrical parameters for the three turbine configurations and the WEC architecture with two 1-meter-diamter turbines.

|  |  |  |  |
| --- | --- | --- | --- |
| **WEC architecture: D=1 m & 2 turbines** | | | |
|  | Varying-chord monoplane | Counter-rotating biplane | Variable-pitch monoplane |
|  | 3 | 3 | 3 |
|  | 0.41 | 0.34 | 0.41 |
|  | 0.46 | 0.55 | 0.46 |
|  | 0.6 | 0.55 | 0.6 |
|  | 0.554 | 0.56 (+ 2%) | 0.647 (+ 17%) |

The optimal number of blades results the same regardless of the turbine configuration and tend to go to the lowest possible value. However, the solidity, both in the hub and the tip, is reasonably high in all configurations. The combination of a low number of blades but high solidity values results in massive blades that can be problematic in terms of structural integrity. In addition, in the case of the variable-pitch configuration, these massive blades are a significant handicap for the design of the control mechanism that allows for pitch angle variations. However, the inclination of the optimiser to benefit the turbine arrangements with a low number of blades is consistent with the formulation of the BEM-based design tool described in Section 3.1. This design tool computes the interaction effects among the different blades using the interference factor described in Equation (X), which is defined as a function of the inverse of the distance between blades. This distance is reduced as the number of blades increases and, thus, the interaction, and the losses caused by these interactions, increase. Therefore, if the final fitness does not drop dramatically for turbines with a higher number of blades, a turbine with at least five blades will be favoured.

Finally, the final fitness scores or overall stochastic efficiency for each turbine configuration is also presented in Table 2. The counter-rotating biplane shows to improve the overall efficiency of the varying-chord monoplane in about 2%,while the variable-pitch monoplane configuration improves that overall efficiency in about 17%, showing a significant improvement compared to the other two configurations. This is due to the higher efficiency peak and the broad-band response of the variable-pitch configuration, as illustrated in Figure 8. The counter-rotating biplane configuration also shows a broad-band response, but the whole curve is shifted towards higher pressure flows, which results in a significantly poor performance in sea-states with low power flows. Table 3 illustrates this issue with the stochastic efficiency values corresponding to each sea-state:

Table 3: The sea-state by sea-state performance of the three different turbine configurations for the WEC architecture with two 1-meter-diameter turbines.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **WEC architecture: D=1 m & 2 turbines** | | | | | | |
|  | Varying-chord | Counter-rotating | | | Variable-pitch | |
|  | 0.063 | 0.069 | 8.703 | 0.064 | | 0.949 |
|  | 0.066 | 0.073 | 10.518 | 0.068 | | 2.896 |
|  | 0.069 | 0.075 | 8.853 | 0.069 | | 0.290 |
|  | 0.055 | 0.059 | 7.664 | 0.053 | | -2.920 |
|  | 0.048 | 0.052 | 8.977 | 0.048 | | 0.418 |
|  | 0.078 | 0.081 | 3.732 | 0.104 | | 33.462 |
|  | 0.044 | 0.045 | 4.368 | 0.055 | | 25.977 |
|  | 0.046 | 0.048 | 3.712 | 0.057 | | 24.672 |
|  | 0.028 | 0.029 | 4.317 | 0.044 | | 57.194 |
|  | 0.010 | 0.010 | -0.021 | 0.016 | | 62.500 |
|  | **0.505** | **0.539 (+ 6.7%)** | | | **0.577 (+ 14%)** | |

Hence, the same exercise presented here for the WEC architecture with two 1-meter-diameter turbines can also be carried for other WEC architectures defined in Table 1. The following Figure summarise the results obtained for the other three WEC architectures:

|  |
| --- |
|  |
|  |
| Figure 16: Stochastic efficiency curves for the three turbine configurations and the rest of WEC architectures. |

Similarly to the initial WEC architecture with two 1-meter-diameter turbines, the variable-pitch monoplane turbine seems to be dominant in all the WEC architectures, showing stochastic curves with higher peak and broader response. However, for the WEC architecture with two 0.8-meter-diameter turbines, the variable-pitch monoplane configuration requires rotational speeds that are over the pre-defined rotational speed range (). Therefore, as described in Section 3.2 and Figure 7, the stochastic efficiency for that specific sea-state is given a null value (). Therefore, the final score of the variable-pitch configuration for that specific WEC architecture drops significantly. Table 4 presents all the geometrical characteristics and the final performance scores for all the analysed WEC architecture and turbine configuration combinations. Note that the percentage values in brackets show the overall efficiency variations with respect to the baseline varying-chord monoplane configuration in each WEC architecture.

Table 4: Main geometrical characteristics and final performance score for all the WEC architecture and turbine configurations.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **D=1 m & 2 turbines** | | | **D=1 m & 3 turbines** | | |
| Varying-chord | Counter-rotating | Variable-pitch | Varying-chord | Counter-rotating | Variable-pitch |
|  | 3 | 3 | 3 | 3 | 3 | 3 |
|  | 0.41 | 0.34 | 0.41 | 0.54 | 0.35 | 0.54 |
|  | 0.46 | 0.55 | 0.46 | 0.41 | 0.46 | 0.41 |
|  | 0.6 | 0.5 | 0.6 | 0.75 | 0.7 | 0.75 |
|  | **0.505** | **0.539 (+ 7%)** | **0.577 (+ 14%)** | **0.602** | **0.591 (- 2%)** | **0.643 (+ 7%)** |
|  | **D=0.8 m & 2 turbines** | | | **D=0.8 m & 3 turbines** | | |
| Varying-chord | Counter-rotating | Variable-pitch | Varying-chord | Counter-rotating | Variable-pitch |
|  | 3 | 3 | 3 | 3 | 3 | 3 |
|  | 0.14 | 0.22 | 0.14 | 0.2 | 0.26 | 0.2 |
|  | 0.51 | 0.33 | 0.51 | 0.58 | 0.47 | 0.58 |
|  | 0.4 | 0.55 | 0.4 | 0.4 | 0.45 | 0.4 |
|  | **0.472** | **0.53 (+ 13%)** | **0.273 (- 42%)** | **0.592** | **0.599 (+ 1%)** | **0.65 (+ 10%)** |

As a conclusion, among all the analysed combinations, the WEC architecture with two 1-meter-diameter variable-pitch monoplane turbines highlighted in light green in Table 4, is shown to be the optimal one, although very close to the architecture with 3 three 1-meter-diameter variable-pitch monoplane turbines.

* 2 turbines instead of 3 turbines:
  + hub-to-tip ratio of the latter is significantly larger with 3 turbines () pro
  + The more turbines, the higher O&M costs (more elements to fail/break and repair) pro
  + Higher controllability with 3 turbines con
  + Lower dependability on the failures con
* 3 blades 🡪 5 blades : a great improvement in terms of structural integrity and relatively low efficiency loss (which once accurately analysed with higher fidelity models and experimental campaigns may be even lower)
* CFD- and experimental-informed BEM-tool incorporated into the GA-based optimisation approach

Annex 2- Genetic algorithm optimization tool description

**Basic notions on optimization:**

The optimization procedure described in 3.2, whose results have been presented in 3.3, relies on a so-called genetic algorithm (GA) for its application. The purpose of the present section is to provide the fundamental notions underlying a GA procedure. For the sake of conciseness, the explanation is limited to drafting a conceptual sketch of the procedure, with further subtleties and notions being available in (cite) for the interested reader.

An optimization procedure can be understood as a problem-solving technique whereby a known model, together with the specification of a desired output, leads to the task of finding the inputs that provide such an output. Usually, however, optimization problems are subjected to constraint satisfactions. The desired output is generally represented as a fitness function that scores the different potential configurations that can be achieved by the combinatorics of the input parameters, and is a function that tends to be defined in a monotonously increasing manner; a constraint, besides, represents a binary evaluation (a Boolean statement) that indicates whether a user-set requirement holds or not. The relationship between the fitness function and the constraint specification is to be understood in terms of trade-off dynamics, meaning that constraints tend to discard configurations for which the fitness function acquires higher values overall. On this respect, the optimization problem defined in 3.2 corresponds to a so-called constrained optimization problem (COP). A COP is basically a search problem in which the parametric space is restricted somehow, with those restrictions stemming from predefined constraint definitions.

**Evolutionary computing:**

There are several optimization algorithms that can be used for solving a COP. The GA approach employed in this work finds its inspiration in the notion of evolutionary computing (EC), which draws its ideas from the process of natural evolution. The main metaphor that serves for understanding the working principles of EC-based algorithms is Darwinian evolution, whereby a trial-and-error process results in the determination of the fittest individual of a species within a given environment. Such trial-and-error basis leads to describing the EC-based algorithms, and the GA approach specifically, as heuristic COP-solving techniques.

On such grounds, it is usual to find biology- and genetics-related notions when describing an EC-based algorithm. The term “genetic” is a key concept that points towards such nomenclature, but it is not the only one: adaptive landscape, genome, mutation, crossover, fitness, genotype, phenotype, allele, pleitropy, polygeny…are notions that populate the basic textbooks versing on GA (cite). For the purposes of the present work, it suffices to introduce a minimal set of those concepts, the ones that are required for understanding the building blocks of a standard GA workflow.

However, before proceeding with the actual description of a GA, it is deemed necessary to mention why EC-based problem-solving techniques are becoming commonplace in computer science and, consequently, in fields such as mathematics, economics or, for mentioning the present case, in engineering. The main technical motivation underlying the development of EC-based algorithms is the growing demand for problem-solving automation, given the increased rate of the research and development capacity that results in a decrement of the time available for thorough problem analysis and tailored algorithm design. This decrement runs in parallel to the increasing complexity of the problems to be solved. Those two trends require robust algorithms with acceptable performance, i.e. codes that are applicable to a wide range of problems and without needing much tailoring for particular applications, while at the same time delivering optimal solutions within a reasonable computation time. EC-based algorithms match those needs acceptably.

Taking it to the design optimization problem of Wells turbines described in 3.2, the main code blocks that perform the aerodynamic analysis of a specific turbine configuration and provide its corresponding stochastic curves under given sea-states are those provided in 3.1. Beyond such computing algorithms, the optimization problem does not rely on any additional code blocks that analyse the suitedness of different turbine configurations. The task of finding the optimal turbine design, given a set of predefined constraints such as the requirement of operating within a range of rotational speeds, is limited to the definition of a fitness function that is embedded in a version of a GA adapted to iteratively scoring different turbine configurations within the paradigm of a search problem.

**Working principles of a GA:**

For solving such a search problem, the basic idea behind EC-based algorithms, and of a GA approach particularly, can be synthesized as follows: given a population of individuals within some environment owning limited resources, competition for those resources causes natural selection, i.e. survival of the fittest. This, in turn, causes a rise in the average fitness of the population.

The understanding of the practical application of such a working principle requires defining certain notions that are alluded to in its statement, as well as other concepts that are needed to grasp the overall idea of a GA. The following list provides the such a minimal set of notions:

* **Population:** it refers to the individuals that the algorithm scores according to a predefined fitness function. In terms of the optimization procedure described in 3.2, each individual corresponds to a specific Wells turbine design, and the population is no more than the overall set of individuals, or the collection of turbine designs that are evaluated.
* **Environment:** it is the background within which the optimization procedure is undertaken. Technically, the notion of parametric-space is better-suited for referring to the environment. Such a space stipulates the parameters that determine the specific configuration of a Wells turbine, and constitute a *de facto* constraint on the set of overall possibilities that are scorable or analysable by the GA algorithm. In the particular case of the procedure in 3.2, the parametric-space comprises four different variables, namely , where the curly braces stand for representing the parametric-space itself, whereas the brackets stand for addressing that each of the parameters can achieve a set of different values. The volume of the parametric-space is the number of all possible combinations of the input variables, and constitutes an indicator of the complexity of an optimization task.
* **Genotype:** it corresponds to the particular set of values that the variables of the parametric-space acquire for a given individual. In other words, and taking it to the practical ground, the particularization of the space for each turbine design, i.e. the collection , for instance. Each of the particular values on the string of numbers is called allele.
* **Phenotype:** it is the instantiated turbine design according to its genotype values. For the , it would be the turbine with 3 blades, a solidity at tip and hub of 0.7, and a hub-to-tip ratio of 0.45, and the corresponding dimensionless stochastic curves and fitness value that would result from its evaluation. It is relevant to discern between the genotype (an ID-like label for each individual) from the phenotype (the instantiation of such a genotype, with two different genotypes being liable to show equivalent aspects on their phenotypes, such as equal fitness values for instance).
* **Generation:** on computational terms, it addresses an iteration of the GA approach within which the individuals of a population undergo competition, mating or crossover and mutation, each virtualized by its corresponding operator. The competition-crossover-mutation cycle models the natural selection process by which the population is assumed to evolve towards the fittest configurations within the provided parametric-space.
* **Competition:** usually, a GA approach considers constant population sizes across generations, which produces a background of limited resources within which a competitive dynamics ensues among the individuals. The competition operator works at the population level, selecting among the individuals those that are supposed to seed the next generation. Such choices are always made relative to what is present in the current population; for instance, the best individual of a given population is chosen to pass to the next generation, whereas the worst one is chosen to be replaced by a new individual.
* **Crossover:** also named parent or mate selection, it distinguishes among individuals based on their fitness and, in particular, to allow the better individuals to become parents of the next generation. An individual is a parent if it has been selected to undergo variation in order to create offspring. Together with the competition mechanism, crossover is responsible for pushing quality improvements. In standard GA approaches, such a crossover is performed probabilistically, with high-fitness individuals having more chances of becoming parents than those with low-fitness.
* **Mutation:** it is a variation operator that creates new individuals from old ones. It is applied to a given genotype and delivers a slightly modified mutant. A mutation operator is stochastic, with its output or child depending on random choices. Mutation is necessary for allowing the connectedness of the parametric-space, meaning that it turns possible searching throughout the overall set of configurations, avoiding to confine the searching algorithm to local maxima just as the sole role of the competition and crossover operators do.
* **Fitness function:** it is the function that provides the score for evaluating each genotype. The ones employed in the present work have been defined in 3.2. The *de facto* constraints imposed by the inherent trimming of the parametric-space are complemented by the user-set constraints at the fitness function level. In other words, the design restrictions of the problem are implemented at the fitness function level.

The synthesis of a standard GA routine is represented in Figure 17.

Figure 17: Standard GA routine

Competition

Mating

Mutation

Parametric-space (PS) definition

No

Yes

Output fittest individual

First random population

Genotype generations

Phenotype instantiations

Fitness calculation

Fitness function definition

User-set constraints

**GA routine**